Open *t-J* chain with boundary impurities

1999 J. Phys. A: Math. Gen. 32 4585

(http://iopscience.iop.org/0305-4470/32/25/302)

J. Phys. A: Math. Gen. **32** (1999) 4585–4591. Printed in the UK PII: S0305-4470(99)02692-X

Open t–J chain with boundary impurities

Gerald Bedürftig and Holger Frahm

Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany

E-mail: bed@itp.uni-hannover.de and frahm@itp.uni-hannover.de

Received 16 March 1999

Abstract. We study integrable boundary conditions for the supersymmetric t–J model of correlated electrons which arise when combining static scattering potentials with dynamical impurities carrying an internal degree of freedom. The latter differ from the bulk sites by allowing for double occupation of the local orbitals. The spectrum of the resulting Hamiltonians is obtained by means of the algebraic Bethe ansatz.

1. Introduction

Impurities in correlated quantum systems have attracted considerable interest recently. In particular, in one spatial dimension exactly solvable models and powerful field-theoretical methods have provided insights into the properties of local perturbations of ideal chain systems [1–7]. Static perturbations such as scattering potentials have a profound effect on the transport properties of quasi one-dimensional (1D) structures such as quantum wires. Impurities with internal degrees of freedom, e.g. a localized magnetic moment in the Kondo problem, may be screened due to resonances with the electrons in the 1D correlated host.

In the framework of the quantum inverse scattering method (QISM) [8] the construction of integrable models for such systems is based on inhomogeneous vertex models constructed from solutions to a Yang–Baxter equation (YBE). Such inhomogeneities were embedded into periodic chains by Andrei and Johannesson for the Heisenberg model [9] and later into various models including the supersymmetric t–J model of interacting electrons by various authors [10–14]. A direct consequence of this way of construction is the lack of backscattering at the impurities [15]. Consideration of such impurities in a more general field-theoretical approach has led to the conclusion that the interactions in the integrable models are fine tuned to a fixed point which is unstable under renormalization flow [4]. As a consequence, the integrable inhomogeneities lack the characteristic properties of a generic potential scatterer in a 1D system with repulsive interactions, which has been found to drive the open-chain fixed point, leading to a vanishing of the conductivity [3].

This can be overcome by combination of these integrable inhomogeneities with a real boundary. Again, the construction of such models is possible within the QISM from solutions to the reflection equations (RE) [16, 17] imposing consistency conditions on the possible boundary conditions for a given bulk system. For the t–J model the simplest such (c-number) solutions of the RE correspond to boundary chemical potentials and boundary magnetic fields respectively [18, 19]. Combining these boundary matrices with solutions of the YBE one can derive dynamical boundary impurity models; Heisenberg models [20, 21] and Kondo-type impurities [1, 22, 23] coupled to correlated electron systems have been studied in this way.

0305-4470/99/254585+07$19.50

Here, we construct the most general boundary impurities that can be realized within this approach by combination of the known static boundary fields for the supersymmetric t–J model with a dynamical impurity allowing for double occupancy of the electronic orbital at its site. This four-state impurity alone, has been studied previously for periodic chains and open ones with reflecting ends [11, 12, 24]. The resulting boundary terms are characterized by the boundary field and, in addition, by a real parameter characterizing the four-dimensional typical representation of the graded Lie algebra $gl(2|1)$ realized on the Hilbert space of the impurity and by its coupling strength to the host system which is controlled by a shift in the spectral parameter of the corresponding vertex. Using both boundary chemical potentials and boundary magnetic fields this leads to two three-parametric families of boundary terms. Further models are obtained by application of the 'projecting method' introduced recently [25]. Finally, we present the Bethe ansatz equations (BAE) determining the spectra of these impurity models.

2. Algebraic construction

Following [16, 17] the classification of integrable boundary conditions within the QISM is based on representations of two algebras $\mathcal{T}_\pm$ defined in terms of REs. For $\mathcal{T}_-$ this equation reads:

$$\mathcal{R}^{12}(\lambda-\mu)\overset{1}{\mathcal{T}}_-(\lambda)\mathcal{R}^{21}(\lambda+\mu)\overset{2}{\mathcal{T}}_-(\mu)=\overset{2}{\mathcal{T}}_-(\mu)\mathcal{R}^{12}(\lambda+\mu)\overset{1}{\mathcal{T}}_-(\lambda)\mathcal{R}^{21}(\lambda-\mu) \tag{2.1}$$

with $\overset{1}{\mathcal{T}}_- = \mathcal{T}\otimes I$ and $\overset{2}{\mathcal{T}}_- = I\otimes\mathcal{T}\dagger$. The algebra $\mathcal{T}_+$ is related to $\mathcal{T}_-$ by an automorphism. Representations of $\mathcal{T}_\pm$ determine the boundary terms in the Hamiltonian at the left (right) end of the chain. Since these can be chosen independently, it is sufficient to consider solutions of (2.1) to obtain a classification of the possible boundary impurities.

The matrix $\mathcal{R}$ in (2.1) solves the quantum YBE:

$$\mathcal{R}^{12}(\lambda)\mathcal{R}^{13}(\lambda+\mu)\mathcal{R}^{23}(\mu)=\mathcal{R}^{23}(\mu)\mathcal{R}^{13}(\lambda+\mu)\mathcal{R}^{12}(\lambda) \tag{2.2}$$

where the superscripts denote the spaces in the tensor product $V_1\otimes V_2\otimes V_3$ in which $\mathcal{R}^{ij}$ acts nontrivially.

For the t–J model this $\mathcal{R}$-matrix is given by:

$$(\mathcal{R}^{12}(\lambda))^{j_1 j_2}_{i_1 i_2}=\frac{\lambda}{\lambda+\mathrm{i}}\delta^{j_1}_{i_1}\delta^{j_2}_{i_2}+\frac{\mathrm{i}}{\lambda+\mathrm{i}}\Pi^{j_1 j_2}_{i_1 i_2} \tag{2.3}$$

with the graded permutation operator $\Pi^{cd}_{ab}=\delta^d_a\delta^b_c(-1)^{[a][b]}$, $[a]\in\{0,1\}$ denoting the grading of the basis states. The c-number solutions of the RE (2.1) corresponding to this $\mathcal{R}$-matrix can be classified [18], below we use the diagonal ones

$$K^p_-=\begin{pmatrix}1 & & \\ & 1 & \\ & & -\frac{p\lambda+\mathrm{i}}{p\lambda-\mathrm{i}}\end{pmatrix}\qquad K^h_-=\begin{pmatrix}-\frac{h\lambda+\mathrm{i}}{h\lambda-\mathrm{i}} & & \\ & 1 & \\ & & 1\end{pmatrix} \tag{2.4}$$

corresponding to a boundary chemical potential p and a boundary magnetic field h (in combination with a chemical potential), respectively [19].

To construct boundary impurities carrying internal degrees of freedom we combine the matrices from (2.4) with an integrable impurity which has been considered previously in a periodic chain [11, 12]. In the QISM this impurity is characterized by the following $\mathcal{L}$-matrix:

$$\mathcal{L}^{34}(\lambda)=\frac{\lambda-\mathrm{i}(\frac{\alpha}{2}+1)}{\lambda+\mathrm{i}(\frac{\alpha}{2}+1)}+\frac{\mathrm{i}}{\lambda+\mathrm{i}(\frac{\alpha}{2}+1)}\tilde{\mathcal{L}}\qquad \tilde{\mathcal{L}}=\begin{pmatrix}1-n_\uparrow & -S^- & Q_\uparrow\\ -S^+ & 1-n_\downarrow & Q_\downarrow\\ Q^\dagger_\uparrow & Q^\dagger_\downarrow & \alpha+2-n\end{pmatrix}. \tag{2.5}$$

† For the t–J model considered here, these tensor products carry a grading and we have to use a graded version of the QISM. For details see, for example, [26].

Here $n = \sum_{\sigma=\uparrow,\downarrow} n_\sigma = \sum_\sigma c_\sigma^\dagger c_\sigma$ and $\vec{S} = \frac{1}{2} c_\alpha^\dagger \vec{\sigma}_{\alpha\beta} c_\beta$ are the electron number and spin operators on the impurity site expressed in terms of canonical fermionic creation and annihilation operators. The Q_σ are the fermionic generators of $gl(2|1)$ in this representation which can be expressed in terms of projection operators (the so-called 'Hubbard operators') $\boldsymbol{X}^{ab} = |a\rangle\langle b|$ with $a, b = \uparrow, \downarrow, 2, 0$:

$$Q_\sigma = \sqrt{\alpha+1}\boldsymbol{X}^{0\sigma} - 2\sigma\sqrt{\alpha}\boldsymbol{X}^{-\sigma 2} \tag{2.6}$$

with $\sigma = \pm\frac{1}{2}$ corresponding to $\sigma = \uparrow, \downarrow$.

$\mathcal{L}^{34}$ acts on a four-dimensional quantum space and satisfies the intertwining relation:

$$\mathcal{R}^{12}(\lambda-\mu)(\mathcal{L}^{34}(\lambda)\otimes\mathcal{L}^{34}(\mu)) = (\mathcal{L}^{34}(\mu)\otimes\mathcal{L}^{34}(\lambda))\mathcal{R}^{12}(\lambda-\mu). \tag{2.7}$$

Following Sklyanin [17], operator-valued matrix solutions of the RE are obtained by 'dressing' a c-number solution $K_-(\lambda)$ of the RE with $\mathcal{L}^{34}$, i.e. considering the product $\mathcal{T}_-(\lambda) = \mathcal{L}^{34}(\lambda+t)K_-(\lambda)\left(\mathcal{L}^{34}(-\lambda+t)\right)^{-1}$ in matrix space (a shift t of the spectral parameter λ is consistent with the intertwining relation (2.7)). To construct an integrable chain with this impurity placed on site one, this reasoning is iterated with the $\mathcal{L}$-operators for the t–J model, i.e. $\mathcal{L}_n = \mathcal{R}^{0n}$, $n = 2, \ldots, L$ resulting in

$$\mathcal{T}_-(\lambda) = \mathcal{L}_L(\lambda)\ldots\mathcal{L}_2(\lambda)\mathcal{L}_1^{34}(\lambda+t)K_-(\lambda)(\mathcal{L}_1^{34}(-\lambda+t))^{-1}(\mathcal{L}_2(-\lambda))^{-1}\ldots(\mathcal{L}_L(-\lambda))^{-1}. \tag{2.8}$$

The integrable model is now defined through the transfer matrix

$$\tau(\lambda) = \mathrm{str}_0[K_+(\lambda)\mathcal{T}_-(\lambda)]. \tag{2.9}$$

$\mathrm{str}_0(M) = \sum_a (-1)^{[a]} M_{aa}$ is the (graded) supertrace taken in matrix space. Since the purpose of this paper is the classification of integrable boundary terms obtained in this class we restrict ourselves to the simplest case of $K_+(\lambda) \equiv \mathbf{1}$ as a representation of the algebra $\mathcal{T}_+$ which corresponds to a reflecting left boundary of the chain. Then, the Hamiltonian is obtained by differentiation of the transfer matrix with respect to the spectral parameter:

$$H \propto \mathrm{i}\frac{\partial}{\partial\lambda}\tau(\lambda)|_{\lambda=0}. \tag{2.10}$$

This leads to the following Hamiltonian of the quantum chain

$$\begin{aligned}\mathcal{H} = -\mathcal{P}\Bigg(\sum_{j=2}^{L-1}\sum_\sigma c_{j,\sigma}^\dagger c_{j+1,\sigma} + c_{j+1,\sigma}^\dagger c_{j,\sigma}\Bigg)\mathcal{P} \\ +2\sum_{j=2}^{L-1}\left[\vec{S}_j\vec{S}_{j+1} - \frac{n_j n_{j+1}}{4} + \frac{1}{2}(n_j + n_{j+1})\right] + \frac{4}{4t^2+(\alpha+2)^2}\mathcal{H}_b^{p,h}\end{aligned} \tag{2.11}$$

where $\mathcal{P}$ projects out double occupancies on the bulk sites, and $\vec{S}_j$, n_j are the electronic spin and number operators on site j defined as above. The boundary terms $\mathcal{H}_b$ depend on the choice of the boundary matrix, after a unitary transformation $\mathcal{H}_b^p$ is given by

$$\begin{aligned}\mathcal{H}_b^p = -p\left(t^2 + \frac{(\alpha+2)^2}{4}\right) + \{1 + p(\alpha+1)\}n_1 + \left\{1+\alpha+p\left(t^2+\frac{\alpha^2}{4}\right)\right\}n_2 \\ +\{1+p\alpha\}\left\{2\vec{S}_1\vec{S}_2 - \frac{n_1 n_2}{2}\right\} + p\{n_2 - 2\}\boldsymbol{X}_1^{22} \\ -\sqrt{\alpha+1}t_0\{\boldsymbol{X}_2^{\uparrow 0}\boldsymbol{X}_1^{0\uparrow} + \boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{0\downarrow} + \text{h.c.}\} \\ -\sqrt{\alpha}t_2\{\boldsymbol{X}_2^{\uparrow 0}\boldsymbol{X}_1^{\downarrow 2} - \boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{\uparrow 2} + \text{h.c.}\}\end{aligned} \tag{2.12}$$

where $t_0 = \sqrt{([1+\frac{p\alpha}{2}]^2+p^2t^2)}$ and $t_2 = \sqrt{([1+p(\frac{\alpha}{2}-1)]^2+p^2t^2)}$. Note that the representation of $gl(2|1)$ entering (2.5) does allow for double occupancy on the first site.

Similarly, we obtain the following boundary operator when considering the K_-^h-matrix:

$$\begin{aligned}\mathcal{H}_b^h = {} & n_1+(\alpha+1)n_2-h(1+\alpha)n_{1\uparrow}-h\left(t^2+\frac{\alpha^2}{4}\right)n_{2\uparrow}\\ & +(\alpha h-1)n_{1\uparrow}n_{2\downarrow}-n_{1\downarrow}n_{2\uparrow}+h(1-n_{2\uparrow})\boldsymbol{X}_1^{22}\\ & +\sqrt{\alpha}(1+h)\{\boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{\uparrow 2}+\text{h.c.}\}-\sqrt{\alpha+1}\{\boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{0\downarrow}+\text{h.c.}\}\\ & -t_c[\sqrt{\alpha}\,\boldsymbol{X}_2^{\uparrow 0}\boldsymbol{X}_1^{\downarrow 2}+\sqrt{\alpha+1}\,\boldsymbol{X}_2^{\uparrow 0}\boldsymbol{X}_1^{0\uparrow}+S_1^+S_2^-+\text{h.c.}]\end{aligned} \tag{2.13}$$

with $t_c=\sqrt{(1-\frac{\alpha h}{2})^2+t^2h^2}$.

The models constructed above can be solved using the algebraic Bethe ansatz. Starting from the completely filled, fully polarized state, i.e. doubly occupied impurity site one and all other sites are occupied by a spin-↑ electron, we find that the spectrum of $\mathcal{H}$ is determined by the solutions of the BAE:

$$\begin{aligned}B_h e_1^{2(L-1)}(\lambda_k) = {} & \prod_{j\neq k}^{M_s} e_2(\lambda_k-\lambda_j)e_2(\lambda_k+\lambda_j)\prod_{\ell=1}^{M_c} e_{-1}(\lambda_k-\vartheta_\ell)e_{-1}(\lambda_k+\vartheta_\ell)\\ & \times\prod_{j=1}^{M_s} e_{-1}(\vartheta_\ell-\lambda_j)e_{-1}(\vartheta_\ell+\lambda_j) = B_p(\vartheta_\ell)e_\alpha(\vartheta_\ell+t)e_\alpha(\vartheta_\ell-t)\end{aligned} \tag{2.14}$$

where $e_n(x)=\frac{x+\mathrm{i}n/2}{x-\mathrm{i}n/2}$, $M_c=L+1-N_e$, $M_s=L-N_\uparrow$ and boundary phase shifts

$$\begin{aligned}B_h(\lambda) &= \begin{cases}1 & \text{for}\quad K_-^p\\ -e_{-1-\frac{2}{h}}(\lambda) & \text{for}\quad K_-^h\end{cases}\\ B_p(\vartheta) &= \begin{cases}-e_{\frac{2}{p}-2}(\vartheta) & \text{for}\quad K_-^p\\ 1 & \text{for}\quad K_-^h.\end{cases}\end{aligned} \tag{2.15}$$

The energy of the corresponding Bethe state is then given by the expression

$$E = E_b^{p,h}+2(L-2)-\sum_{j=1}^{M_s}\frac{1}{\lambda_j^2+\frac{1}{4}} \tag{2.16}$$

with $E_b^p=(4\alpha+8)/(4t^2+(\alpha+2)^2)$ and $E_b^h=(4\alpha+8)/(4t^2+(\alpha+2)^2)-h$.

As for the periodic and the open t–J model, the zero-temperature ground state and the low-lying charged and magnetic excitations are characterized by real solutions for the λ and ϑ rapidities of the BAE.

3. Projecting method

Recently, it has been realized that new integrable boundary Hamiltonians may be obtained after fine tuning of the parameters characterizing the boundary and impurity, respectively [25] by projection onto an invariant subspace. An important application of this procedure is a Kondo spin coupled to the t–J model [22, 23, 27]. To apply the projecting method one has to find a decomposition of the Hilbert space of the impurity $\mathbf{H}=\mathbf{H}_1\oplus\mathbf{H}_2$ and fine tune the parameters in $\mathcal{T}_-$ such that one of the following conditions is satisfied ($\Pi_{1,2}$ are projectors onto $\mathbf{H}_{1,2}$)

$$\Pi_1\mathcal{T}_-\Pi_2=0 \qquad \text{or} \qquad \Pi_2\mathcal{T}_-\Pi_1=0. \tag{3.1}$$

Starting from K_-^p, we find that the decomposition $\mathbf{H} = \{\uparrow, \downarrow, 0\} \oplus \{2\}$ is possible for $t = \bar{t} \equiv \mathrm{i}(\frac{\alpha}{2} - 1 + \frac{1}{p})$; the resulting Hamiltonians do not lead to new models. For $\mathcal{H}_2(\alpha, p, \bar{t})$, the impurity is a scalar one and the model reduces to an open t–J model with boundary chemical potential at site two. The other possible projection, $\mathcal{H}_{\uparrow,\downarrow,0}(\alpha, p, \bar{t})$, is a simple reparametrization of $\mathcal{H}_{\uparrow,\downarrow,0}(\alpha = 0, p, t)$.

Choosing $t = \tilde{t} \equiv \mathrm{i}(\frac{\alpha}{2} + \frac{1}{p})$ the condition (3.1) is satisfied for the decomposition $\mathbf{H} = \{2, \uparrow, \downarrow\} \oplus \{0\}$. Again $\mathcal{H}_0(\alpha, p, \tilde{t})$ is just an open t–J model with boundary chemical potential at the second site. Projecting onto the second subspace, however, we find a two-parametric Hamiltonian:

$$\begin{aligned}\mathcal{H}_{2,\uparrow,\downarrow} = &-(a+b+2) + \frac{(a+b+2)^2}{a+b+1}n_1 + \frac{a+b+2}{3+2a+b}n_2 \\ &+\frac{(a+1)(a+b+2)^2}{(a+b+1)(3+2a+b)}\left\{2\vec{S}_1\vec{S}_2 - \frac{n_1 n_2}{2}\right\} \\ &+\frac{(a+b+2)^3}{(a+b+1)(3+2a+b)}\{n_2 - 2\}\boldsymbol{X}_1^{22} \\ &-\frac{(a+b+2)^2}{(a+b+1)(3+2a+b)}\sqrt{ab}\{\boldsymbol{X}_2^{\uparrow 0}\boldsymbol{X}_1^{\downarrow 2} - \boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{\uparrow 2} + \text{h.c.}\}. \end{aligned} \tag{3.2}$$

The Hamiltonian $\mathcal{H}_{2,\uparrow,\downarrow}$ may be constructed with aid of $\mathcal{L}^{34}(\alpha = -1)$ and choosing the remaining two parameters as $t = -\frac{\mathrm{i}}{2}(a+b)/(a+b+2)$ and $p = (a+b+2)/(3+2a+b)$. This $\mathcal{L}$-matrix corresponds to the one obtained by using the dual of the fundamental three-dimensional representation of the algrbra $gl(2|1)$. This solution of the YBE has been used in [28, 29] to construct periodic t–J models with impurities and usual t–J sites, alternating. Choosing either $a = 0$ or $b = 0$ a further projection is possible: $\mathbf{H} = \{2\} \oplus \{\uparrow, \downarrow\}$. $\mathcal{H}_2$ corresponds to a boundary chemical potential at the second site. Substituting $b = -\frac{a}{1+a}$ in $\mathcal{H}^{a=0}_{\uparrow,\downarrow}$, one obtains $\mathcal{H}^{b=0}_{\uparrow,\downarrow}$. The resulting Hamiltonian $\mathcal{H}^{a=0}_{\uparrow,\downarrow}$ can be identified with a spin-$\frac{1}{2}$ Kondo impurity introduced in [22].

Considering K_-^h, only one decomposition $\mathbf{H} = \{2, \uparrow\} \oplus \{0, \downarrow\}$ satisfying (3.1) is possible. Choosing $t = \mathrm{i}(\frac{\alpha}{2} - \frac{1}{h})$ we find two new boundary Hamiltonians, namely

$$\begin{aligned}\mathcal{H}_{2,\uparrow} = &\left[\frac{h^2}{(h+1)(h(\alpha+1)-1)}\right]\left\{n_1 + (\alpha+1)(n_2 - hn_{1\uparrow}) + n_{2\uparrow}\frac{1-\alpha h}{h}\right. \\ &\left.+(\alpha h - 1)n_{1\uparrow}n_{2\downarrow} + h(1-n_{2\uparrow})\boldsymbol{X}_1^{22}\right\} + \frac{\sqrt{\alpha}h^2}{h(\alpha+1)-1}\{\boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{\uparrow 2} + \text{h.c.}\} \end{aligned} \tag{3.3}$$

and

$$\begin{aligned}\mathcal{H}_{\downarrow,0} = &\left[\frac{h^2}{(h+1)(h(\alpha+1)-1)}\right]\left\{n_1 + (\alpha+1)n_2 + \frac{1-\alpha h}{h}n_{2\uparrow} - n_{1\downarrow}n_{2\uparrow}\right. \\ &\left.-\sqrt{\alpha+1}\{\boldsymbol{X}_2^{\downarrow 0}\boldsymbol{X}_1^{0\downarrow} + \text{h.c.}\}\right\}. \end{aligned} \tag{3.4}$$

Both (3.3) and (3.4) resemble certain features of the Anderson model for a local orbital coupled to a correlated host.

The BAE for the projected Hamiltonians coincide with the ones for the original model (2.14), *provided* that the reference state used in their construction, i.e. the state $|2\rangle$ in the impurity Hilbert space is not projected out. Hence, to obtain the spectrum of $\mathcal{H}_{\downarrow,0}$ one has to use different BAE obtained for a suitable pseudo-vacuum. Alternatively, one may consider

solutions of (2.14) *after* adding the complex solutions corresponding to bound states (see e.g. [7, 30]):

$$\lambda = \mathrm{i}\left(\frac{1}{2} + \frac{1}{h}\right) \qquad \text{and} \qquad \vartheta = \frac{\mathrm{i}}{h}. \tag{3.5}$$

This results in the following set of BAE for for the Hamiltonian (3.4)

$$-e_{3+2/h}(\lambda_k)e_1^{2(L-1)}(\lambda_k) = \prod_{j\neq k}^{M_s} e_2(\lambda_k - \lambda_j)e_2(\lambda_k + \lambda_j)\prod_{\ell=1}^{M_c} e_{-1}(\lambda_k - \vartheta_\ell)e_{-1}(\lambda_k + \vartheta_\ell)$$
$$\times \prod_{j=1}^{M_s} e_{-1}(\vartheta_\ell - \lambda_j)e_{-1}(\vartheta_\ell + \lambda_j) = e_{2\alpha-2/h}(\vartheta_\ell)e_{2+2/h}((\vartheta_\ell) \tag{3.6}$$

where $M_c = L - N_e$ and $M_s = L - 1 - N_\uparrow$. The energy of the corresponding Bethe state with spectral parameters $\{\lambda_j\}$ and $\{\vartheta_\ell\}$ is given by

$$E = \frac{h}{h(\alpha+1)-1} + 2(L-2) - \sum_{j=1}^{M_s} \frac{1}{\lambda_j^2 + \frac{1}{4}}. \tag{3.7}$$

4. Summary

Starting from a particular solution of the intertwining relation (2.7) built from a four-dimensional representation of the graded Lie algebra $gl(2|1)$ and diagonal c-number solutions (2.4) of the RE, we have constructed supersymmetric t–J models with integrable boundary impurities of Anderson- or Kondo-type, i.e. with an internal degree of freedom. Due to non-zero boundary potentials, the resulting boundary terms break the supersymmetry of the model; the most general ones which can be constructed this way are given in equations (2.12) and (2.13). The presence of the boundary allowed for fine tuning of these potentials to project the model to a remaining invariant subspace. In most cases this projection led to models which had been constructed directly before. The Anderson-type impurities described by equations (3.3) and (3.4), however, are novel.

The spectra of these models have been obtained by means of the algebraic Bethe ansatz. Furthermore, having the exact dependence of the ground state energy on the parameters defining the impurity allows for the computation of *local* correlations which, usually, are not easily accessible from the Bethe ansatz solution (see [12, 20] for examples).

Acknowledgments

We would like to thank A A Zvyagin for discussions. This work was supported by the Deutsche Forschungsgemeinschaft under Grant No Fr 737/2–3.

References

[1] Schulz H 1987 *J. Phys. C: Solid State Phys.* **20** 2375
Schulz H 1987 *J. Phys. C: Solid State Phys.* **20** 4999(E)
[2] Göhmann F and Schulz H 1990 *J. Phys.: Condens. Matter* **2** 3841
[3] Kane C L and Fisher M P A 1992 *Phys. Rev.* B **46** 15 233
[4] Sørensen E S, Eggert S and Affleck I 1993 *J. Phys. A: Math. Gen.* **26** 6757
[5] Asakawa H and Suzuki M 1996 *J. Phys. A: Math. Gen.* **29** 7811
[6] Affleck I 1997 *Nucl. Phys. B (Proc. Suppl.)* **58** 35
[7] Eßler F H L and Frahm H 1997 *Phys. Rev.* B **56** 6631

[8] Korepin V E, Bogoliubov N M and Izergin A G 1993 *Quantum Inverse Scattering Method and Correlation Functions* (Cambridge: Cambridge University Press)
[9] Andrei N and Johannesson H 1984 *Phys. Lett.* A **100** 108
[10] Bares P-A 1994 *Preprint* cond-mat/9412011
[11] Bedürftig G, Eßler F H L and Frahm H 1996 *Phys. Rev. Lett.* **77** 5098
[12] Bedürftig G, Eßler F H L and Frahm H 1997 *Nucl. Phys.* B **489** 697
[13] Schlottmann P and Zvyagin A A 1997 *Phys. Rev.* B **55** 5027
[14] Foerster A, Links J and Tonel A *Nucl. Phys.* B to appear
[15] Eckle H-P, Punnoose A and Römer R A 1997 *Europhys. Lett.* **39** 293
[16] Cherednik I V 1984 *Theor. Math. Phys.* **61** 977
[17] Sklyanin E K 1988 *J. Phys. A: Math. Gen.* **21** 2375
[18] González-Ruiz A 1994 *Nucl. Phys.* B **424** 468
[19] Eßler F H L 1996 *J. Phys. A: Math. Gen.* **29** 6183
[20] Frahm H and Zvyagin A A 1997 *J. Phys.: Condens. Matter* **9** 9939
[21] Wang Y 1997 *Phys. Rev.* B **56** 14 045
[22] Wang Y, Dai J-H, Hu Z-N and Pu F-C 1997 *Phys. Rev. Lett.* **79** 1901
[23] Hu Z-N, Pu F-C and Wang Y 1998 *J. Phys. A: Math. Gen.* **31** 5241
[24] Bedürftig G, Frahm H and Zvyagin A A unpublished
[25] Frahm H and Slavnov N A 1999 *J. Phys. A: Math. Gen.* **32** 1547
[26] Eßler F H L and Korepin V E 1992 *Phys. Rev.* B **46** 9147
[27] Zhou H-Q, Ge X-Y, Links J and Gould M D 1999 *Nucl. Phys.* B **546** 779
[28] Abad J and Ríos M 1999 *J. Phys. A: Math. Gen.* **32** 3535
[29] Links J and Foerster A 1999 *J. Phys. A: Math. Gen.* **32** 147
[30] Bedürftig G and Frahm H 1997 *J. Phys. A: Math. Gen.* **30** 4139